Unpacked South Dakota State Mathematics Standards

Purpose: *In order for students to have the best chance of success, standards, assessment, curriculum resources, and instruction must be aligned in focus, coherence, and rigor. Unpacked standards documents are intended to help align instruction to the focus, coherence, and rigor of the South Dakota State Mathematics Standards. The standards have been organized in clusters as they are not so much built from topics, but rather woven out of progressions. Not all content in a given grade is emphasized equally in the mathematics standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting standards will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.*

<table>
<tr><td colspan="2">Domain: Statistics and Probability</td><td>Grade Level: 4th Year</td></tr>
<tr><td colspan="3">HS4.S.MD.A Cluster: Calculate expected values and use them to solve problems.

Students will calculate expected value of a random variable to interpret and make predictions using theoretical probability and empirical data.</td></tr>
<tr><td colspan="3">This is a MAJOR cluster.
Students should spend the large majority of their time (65-85%) on the major work of the grade. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.

S.MD.1. Assign a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
S.MD.2. Calculate the expected value of a random variable; understand that it is the mean of the probability distribution.
S.MD.3. Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; calculate the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.
S.MD.4. Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; calculate the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?</td></tr>
<tr><td colspan="3">Aspects of Rigor for Student Learning: (Conceptual, Procedural, and/or Application)</td></tr>
<tr><td colspan="3">S.MD.1. Assign a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.</td></tr>
<tr><td>Conceptual Understanding</td><td>Procedural Fluency</td><td>Application</td></tr>
<tr><td>Students will determine when to define a random variable.

Students will construct a probability distribution for a random variable.</td><td>Students use appropriate notation for random variables.

Students graph probability distributions.</td><td>Students represent a situation by defining a random variable and graphing the probability distribution.</td></tr>
<tr><td colspan="3">S.MD.2. Calculate the expected value of a random variable; understand that it is the mean of the probability distribution.</td></tr>
<tr><td>Conceptual Understanding</td><td>Procedural Fluency</td><td>Application</td></tr>
<tr><td>Students understand that the expected value is the mean of the probability distribution.

Students can identify the expected</td><td>Students can calculate expected value for a probability distribution.</td><td>Students can use expected value to make predictions for simple situations such as games of chance, dice, and coin tosses.</td></tr>
</table>

value graphically as the “balance point.” Students can interpret expected value as a long-term average value.		
S.MD.3. Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; calculate the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.		
Conceptual Understanding	***Procedural Fluency***	***Application***
Students know that expected value is the mean or long-term average value for a probability distribution.	Students can use appropriate notation to represent a probability distribution for a random variable defined for a sample space with theoretical probabilities. Students can calculate the expected value.	Students can use expected value to interpret and make predictions using theoretical probabilities. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.
S.MD.4. Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; calculate the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?		
Conceptual Understanding	***Procedural Fluency***	***Application***
Students understand that expected value is a result expected over several trials Students understand that expected value is the mean or long-term average value for a probability distribution.	Students can use appropriate notation to represent a probability distribution for a random variable defined for a sample space with empirical probabilities. Students can calculate the expected value.	Students can use expected value to interpret and make predictions using empirical data such as sports statistics, insurance, lottery, or consumer trends.
Enacting the Mathematical Practices - Evidence of Students Engaging in the Practices		

1. **Make sense of problems and persevere in solving them.**
   - Students use a random variable to represent a problem, develop and graph a probability distribution, and calculate expected value.
2. **Reason abstractly and quantitatively.**
   - Students represent problems using random variables and probability distributions. Students interpret expected value as the mean of a probability distribution.
3. **Construct viable arguments and critique the reasoning of others.**
   - Students interpret expected value to make long-term predictions and justify their argument.
4. **Model with mathematics.**
   - Students model with expected value.
5. **Use appropriate tools strategically.**
   - Students know when to define a random variable or calculate expected value.
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Vertical and Horizontal Coherence and Learning Progressions		
Previous Learning Connections	***Current Learning Connections***	***Future Learning Connections***
In middle school and high school (grades 7 and 9), students learned about mean as a measure of center. Students have also learned about probability (grade 10).	Students may use permutations and combinations to calculate probabilities for S.MD.3 Students use expected value when using binomial distributions and margins of error.	Expected value has applications in college-level statistics courses and careers in many industries.

Vocabulary (key terms and definitions)
random variable sample space event probability distribution expected value empirical probability theoretical probability

Relevance, Explanations, and Examples:
Since expected value can be thought of as the long-term average value, it can be used to determine fair value for insurance or games of chance. For example, based on the probabilities of winning certain lottery prizes, fair value for a ticket can be calculated. Fair premiums for health insurance can be calculated in a similar manner. Expected value can also be used to make predictions and strategy decisions in sports. For example, a basketball player could use past data on 2 and 3 point shots to determine which shots should result in more points in a game.