

# Unpacked South Dakota State Mathematics Standards

**Purpose:** In order for students to have the best chance of success, standards, assessment, curriculum resources, and instruction must be aligned in focus, coherence, and rigor. Unpacked standards documents are intended to help align instruction to the focus, coherence, and rigor of the South Dakota State Mathematics Standards. The standards have been organized in clusters as they are not so much built from topics, but rather woven out of progressions. Not all content in a given grade is emphasized equally in the mathematics standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting standards will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.

<b>Domain: Algebra</b>		<b>Grade Level: Algebra 2</b>
<b>A2.A.APR.A Cluster: Understand the relationship between zeros and factors of polynomials</b>		
Learners are exploring the connection of zeros to x-intercepts on a graph and ways to determine the zeros. Factoring, the Remainder Theorem and visual representations are three ways to find zeros of functions that can also yield other information.		
<p><b>**This is a SUPPORTING cluster.</b> Students should spend the large majority of their time (65-85%) on the major work of the grade. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.</p>		
<b>A2.A.APR.A.2 Know and Apply the Remainder Theorem</b>		
<b>A2.A.APR.A.3 Identify Zeros of Polynomials by Factoring</b>		
<p>a. When suitable factorizations are available, use the zeros to construct a rough graph of the function</p> <p>b. When given a graph, use the zeros to construct a possible factorization of a polynomial.</p>		
<b>Aspects of Rigor for Students:</b> (Conceptual, Procedural, and/or Application)		
<b>A2.A.APR.A.2 Know and Apply the Remainder Theorem</b>		
<b>Conceptual Understanding</b>	<b>Procedural Fluency</b>	<b>Application</b>
<p>Learners connect the relationship between the remainder and divisor of an expression. Students recognize that when a remainder is zero the divisor is a factor of the expression and when the remainder is not zero the divisor is not a factor.</p> <p>Example:            Given <math>(6x^3 + 10x^2 - 7x - 4) \div (3x - 1)</math>,  <math>(3x - 1)</math> is not a factor.            Given <math>(6x^3 + 10x^2 - 7x - 1) \div (3x - 1)</math>,  <math>(3x - 1)</math> is a factor.</p>	<p>Learners will be able to use the following methods to divide polynomials:</p> <ul style="list-style-type: none"> <li>• Long division (array method can also be used - see below)</li> <li>• Synthetic division</li> </ul>	
<b>A2.A.APR.A.3 Identify Zeros of Polynomials by Factoring</b>		
<p>a. When suitable factorizations are available, use the zeros to construct a rough graph of the function</p> <p>b. When given a graph, use the zeros to construct a possible factorization of a polynomial</p>		
<b>Conceptual Understanding</b>	<b>Procedural Fluency</b>	<b>Application</b>
Learners will understand the	Learners will be able to:	Learners can apply the concept of

<p>connection between a factor, a solution, and zero of a polynomial function, and its corresponding graph. Students should recognize the connections between a table, graph, and equation.</p> <p>Example: If you know that (3, 0) is a zero of the function, then you know that <math>x = 3</math> is the solution and <math>(x-3)</math> is a factor (and vice versa) and what that visual looks like.</p>	<ul style="list-style-type: none"> <li>Factor polynomials with degrees higher than 2</li> <li>Graph zeros</li> <li>Use the graph to determine possible factors of the function</li> </ul>	<p>zeros to volume problems.</p>
---	---	----------------------------------

### Enacting the Mathematical Practices - Evidence of Students Engaging in the Practices

- 1. Make sense of problems and persevere in solving them.**
- 2. Reason abstractly and quantitatively.**
- 3. Construct viable arguments and critique the reasoning of others.**
  - Students will be able to explain in their own words what it means to factor an expression, what a zero of an equation represents and how it relates to its graph
  - Students will be able to explain how the quotient and remainder of a polynomial division problem are related.
- 4. Model with mathematics.**
  - Students apply this knowledge to real world problems such as volume.
- 5. Use appropriate tools strategically.**
  - Students will know when to use a calculator to determine zeros vs. factoring a problem. Students will know when to use long division vs. synthetic division.
- 6. Attend to precision.**
- 7. Look for and make use of structure.**
  - When given a polynomial function in factored form, students will be able to find the zeros, plot the zeros and then make a sketch of the graph of that is reflective of the function (and its other key features).
- 8. Look for and express regularity in repeated reasoning.**
  - Students connect the representations of functions on the coordinate plane with their zeros and factored forms.

### Vertical and Horizontal Coherence and Learning Progressions

<u>Previous Learning Connections</u>	<u>Current Learning Connections</u>	<u>Future Learning Connections</u>
<p><i>Students are building on their knowledge of zeros and factors of quadratics learned in Algebra 1.</i></p>	<p><i>Students are learning about factoring with polynomials of degrees higher than 2 (perfect cubes, quartics, factor by grouping, etc). Students are also understanding that not all polynomials are factorable, but still can be divided by another polynomial. Students continue to build their understanding of how factored form relates to zeros on a graph. Later in the year, these skills are used in simplifying rational expressions.</i></p>	<p><i>In 4th year math (Pre-Calculus, Calculus, and college level math) students will build on their factoring skills (with rational expressions and trigonometric expressions). Students will also determine zeros of trigonometric functions in subsequent math courses.</i></p>

### Vocabulary (key terms and definitions)

- |  |  |
|--|--|
| <ul style="list-style-type: none"> <li>Factor of a Polynomial</li> <li>Polynomial</li> <li>Remainder</li> <li>Remainder Theorem</li> </ul> | <ul style="list-style-type: none"> <li>Quotient</li> <li>X-intercepts</li> <li>Zero Factor Property</li> <li>Zero of a function</li> </ul> |
|--|--|

**Relevance, Explanations, and Examples:**

Students need to be able to connect between a polynomial equation (function) and its graph (and vice versa).

**This is an example of using an array to divide:**

Given the function  $f(x) = x^3 + 3x^2 - 10x - 24$ . Find the zeros of the function.

- ① Find a real zero from the graph (or could be given)  $x = -4$
- ② Divide out the factor that corresponds to zero  $\Rightarrow x + 4$

	$x^2$	$-x$	$-6$	R
$x$	$x^3$	$-x^2$	$-6x$	0
$+4$	$4x^2$	$-4x$	$-24$	

$$x^3 + 3x^2 - 10x - 24 = (x + 4)(x^2 - x - 6)$$

- ③ Take quotient and factor/solve.

$$x^2 - x - 6 = (x - 3)(x + 2) = 0$$

$$x - 3 = 0$$

$$x = 3$$

$$x + 2 = 0$$

$$x = -2$$

There are three real zeros at  $x = 3, -2, -4$

**Achievement Level Descriptors**

**Cluster: Understand the relationship between zeros and factors of polynomials**

**Concepts and Procedures**

**Level 1:** Students should be able to base arguments on concrete referents such as objects, drawings, diagrams, and actions and identify obvious flawed arguments in familiar contexts.

**Level 2:** Students should be able to find and identify the flaw in an argument by using examples or particular cases. Students should be able to break a familiar argument given in a highly scaffolded situation into cases to determine when the argument does or does not hold.

**Level 3:** Students should be able to use stated assumptions, definitions, and previously established results and examples to test and support their reasoning or to identify, explain, and repair the flaw in an argument. Students should be able to break an argument into cases to determine when the argument does or does not hold.

**Level 4:** Students should be able to use stated assumptions, definitions, and previously established results to support their reasoning or repair and explain

	<p>the flaw in an argument. They should be able to construct a chain of logic to justify or refute a proposition or conjecture and to determine the conditions under which an argument does or does not apply.</p>
--	--