

Unpacked South Dakota State Mathematics Standards

Purpose: In order for students to have the best chance of success, standards, assessment, curriculum resources, and instruction must be aligned in focus, coherence, and rigor. Unpacked standards documents are intended to help align instruction to the focus, coherence, and rigor of the South Dakota State Mathematics Standards. The standards have been organized in clusters as they are not so much built from topics, but rather woven out of progressions. Not all content in a given grade is emphasized equally in the mathematics standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting standards will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.

Domain: Expressions and Equations		Grade Level: 6
6.EE.A Cluster: Apply and extend previous understandings of numbers to the system of rational numbers		
<p>The focus for this cluster is writing and evaluating numerical expressions involving whole number exponents, finding the value of an expression using exponential notation such as $4^2 = 4 \times 4$ or $d^3 = d \times d \times d$, and using the appropriate terminology to explain how to evaluate an expression. Students are applying the properties of operations to generate equivalent expressions including the distributive property to produce equivalent representation.</p>		
<p>**This is a MAJOR cluster. Students should spend the large majority of their time (65-85%) on the major work of the grade. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.</p>		
<p>6.EE.1 Write and evaluate numerical expressions involving whole number exponents (e.g. parentheses, brackets, or braces).</p>		
<p>6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.</p> <ul style="list-style-type: none"> a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as $5 - y$. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms. c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real - world problems. d. Perform arithmetic operations following the order of operations with and without parentheses, including those involving whole - number exponents. 		
<p>6.EE.3 Apply the properties of operations to generate equivalent expressions with an emphasis on the distributive property.</p>		
<p>6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).</p>		
Aspects of Rigor for Student Learning: (Conceptual, Procedural, and/or Application)		
Conceptual Understanding	Procedural Fluency	Application
<p>Understand the concept of exponents and exponential notation. (e.g. $4^2 = 4 \times 4$ or $d^3 = d \times d \times d$) (6.EE.1)</p> <p>Use appropriate terminology to explain how to evaluate an</p>	<p>Evaluate numerical expressions using their knowledge of order of operations from previous years. (6.EE.1)</p> <p>Write a numerical expression that uses addition, subtraction,</p>	

<p>expression. (6.EE.1)</p>	<p>multiplication, division and whole number exponents. (6.EE.1)</p>													
<p>Understand that variables represent unknown quantities. (6.EE.2.a)</p> <p>Understand how to write expressions using variables that represent unknown numbers. (6.EE.2.a)</p> <p>Identify context to write algebraic expressions. (6.EE.2.a)</p>														
<p>Move flexibly between viewing parenthesis groupings as both a single entity (e.g. $8 + 7 = 15$) and also the sum of two terms. (6.EE.2.b)</p> <p>Identify context to write algebraic expressions. (6.EE.2.b)</p>	<p>Translate verbal expressions into numerical expressions. (6.EE.2.b)</p> <p>Translate numerical expressions to verbal expressions. (6.EE.2.b)</p>													
	<p>Perform basic operations. (6.EE.2.c)</p> <p>Substitute the information from a problem into the expression/formula. (6.EE.2.c)</p>	<p>Use information from real world examples to evaluate expressions with variables. (6.EE.2.c)</p> <p>Apply an algebraic expression to formulas such as volume and area. (6.EE.2.c)</p>												
<p>.</p>	<p>Use order of operations to evaluate algebraic expressions. (6.EE.2.d)</p>													
<p>Understand the distributive property extensively. (6.EE.3)</p> <p>Understand the other properties such as commutative, associative and identity properties. (6.EE.3)</p> <table border="1" data-bbox="110 1325 553 1541"> <thead> <tr> <th>Property</th> <th>Addition</th> <th>Multiplication</th> </tr> </thead> <tbody> <tr> <td>Commutative Property</td> <td>You can add in any order $a + b = b + a$ $2 + 4 = 4 + 2 = 6$</td> <td>You can multiply in any order $a \times b = b \times a$ $3 \times 4 = 4 \times 3 = 12$</td> </tr> <tr> <td>Associative Property</td> <td>When you add, you can group the numbers in any combination $a + (b + c) = (a + b) + c$ $1 + (3 + 4) = (1 + 3) + 4$</td> <td>When you multiply, you can group the numbers in any combination $a \times (b \times c) = (a \times b) \times c$ $2 \times (3 \times 5) = (2 \times 3) \times 5$</td> </tr> <tr> <td>Identity Property</td> <td>The sum of zero and any number is the number $a + 0 = a$ $4 + 0 = 4$</td> <td>The product of 1 and any number is the number $a \times 1 = a$ $3 \times 1 = 3$</td> </tr> </tbody> </table> <p>Understand that all of these properties also apply to expressions with variables. (6.EE.3)</p>	Property	Addition	Multiplication	Commutative Property	You can add in any order $a + b = b + a$ $2 + 4 = 4 + 2 = 6$	You can multiply in any order $a \times b = b \times a$ $3 \times 4 = 4 \times 3 = 12$	Associative Property	When you add, you can group the numbers in any combination $a + (b + c) = (a + b) + c$ $1 + (3 + 4) = (1 + 3) + 4$	When you multiply, you can group the numbers in any combination $a \times (b \times c) = (a \times b) \times c$ $2 \times (3 \times 5) = (2 \times 3) \times 5$	Identity Property	The sum of zero and any number is the number $a + 0 = a$ $4 + 0 = 4$	The product of 1 and any number is the number $a \times 1 = a$ $3 \times 1 = 3$		
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<p>Reason that two expressions are equivalent through the use of substitution [e.g. $y + y + y = 3y$ for any y-value and $3(2 + x) = 6 + 3x$ for any x-value] (6.EE.4)</p>	<p>Combine like terms. (6.EE.4)</p>													

Enacting the Mathematical Practices - Evidence of Students Engaging in the Practices

1. **Make sense of problems and persevere in solving them.**
 - Look for meaning in the problems and find effective ways to represent and solve them.
 - Understand what the variable is represented in the problem in front of them stands for in order to make sense of the problem and solve for it. They will be able to explain what the variable represents and how their answer makes sense.
2. **Reason abstractly and quantitatively.**
 - Reason with symbolic representations in equations.
 - Manipulate expressions while keeping equality.
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
 - Model real-life situations with mathematics and use variables to represent two quantities in real world contexts.
 - Model their situations using symbols, tables, and graphs.
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
 - Use appropriate vocabulary and translate between verbal and numerical expressions fluently and accurately.
 - Set up expressions, equations, and/or inequalities that represent the correct interpretation of the problem at hand (e.g. $5 - y$ vs. $y - 5$).
 - State precisely the meaning of variables they use when setting up equations.
7. **Look for and make use of structure.**
 - Apply properties to generate equivalent expressions.
 - Use the structure of the properties to generate the expressions and will need to prove that their expressions are equivalent by using substitution.
 - Interpret the structure of an expression in terms of a context: if a runner is $7t$ miles from her starting point after t hours, what is the meaning of the 7 ?
8. **Look for and express regularity in repeated reasoning.**
 - Look for regularity in a repeated calculation and express it with a general formula.

Vertical and Horizontal Coherence and Learning Progressions

<i>Previous Learning Connections</i>	<i>Current Learning Connections</i>	<i>Future Learning Connections</i>
<p>Learners, in grade 5, have already been taught how to use whole-number exponents to denote powers of 10 (through scientific notation), so they should be familiar with the set-up of exponents.</p> <p>In grade 5, learners have already been taught how to write simplify expressions, but have not yet evaluated the expressions. They have been taught that $(8+7)$ is equivalent to (15) and they should know the commutative and associative property of both addition and multiplication.</p> <p>Learners have learned order of operations without exponents.</p>	<p>Learners have already been taught how to find the greatest common factor of two whole numbers and use the distributive property to express sums of whole numbers [Example: express $36 + 8$ as $4(9 + 2)$]</p> <p>(6.NS.B.4). This will help the learners as they create equivalent expressions (6.EE.A.3) and as they determine and identify equivalent expressions (6.EE.A.4).</p>	<p>In 7th grade, learners will learn to apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. Learners will develop an understanding of operations with rational numbers when working with expressions and linear equations.</p> <p>In 8th grade, students will know and apply the properties of integer exponents to generate equivalent numerical expressions.</p> <p>In high school, students will need to interpret parts of an expression, such as terms, factors, and coefficients.</p>

Vocabulary (Key Terms Used by Teachers and Students in this Cluster):

- | | | |
|------------|------------|---------------------------|
| • Exponent | • Quantity | • Properties |
| • Base | • Quotient | • Multiplicative Identity |

- Numerical Expressions
- Evaluate
- Sum
- Term
- Product
- Factor

- Algebraic Expressions
- Coefficient
- Constant
- Like Terms
- Equivalent Expressions
- Variable

- Distributive Property
- Commutative Property
- Equation
- Expression
- Substitution

Relevance, Explanations, and Examples:

According to the progressions, students have been writing numerical expressions since Kindergarten. Students have been using variables since the Grade 3. In Grade 5 they started using whole number exponents to express powers of 10. In the Grade 6 they start to incorporate whole number exponents into numerical expressions. In Grade 6 they begin to work systematically with algebraic expressions. The students start to interpret the structure of an expression in terms of a context. Though learners should be familiar with parenthesis, this is the first time that they will see brackets and braces. This is the first time that the learners have evaluated expressions with variables. So far, all they have done is set up basic expressions.

Task

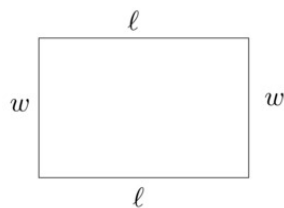
Which of the following expressions are equivalent? Why? If an expression has no match, write 2 equivalent expressions to match it.

- a. $2(x + 4)$
- b. $8 + 2x$
- c. $2x + 4$
- d. $3(x + 4) - (4 + x)$
- e. $x + 4$

Task

The students in Mr. Nolan's class are writing expressions for the perimeter of a rectangle of side length ℓ and width w . After they share their answers, the following expressions are on the board:

- Sam: $2(\ell + w)$
- Joanna: $\ell + w + \ell + w$
- Kiyoo: $2\ell + w$
- Erica: $2w + 2\ell$



Which of the expressions are correct and how might the students have been thinking about finding the perimeter of the rectangle?

Distribution Example:

one combo meal

$$3 \left(2 \text{ 🌮} + \text{ 🥤} \right) = 6 \text{ 🌮} + 3 \text{ 🥤}$$

$$3(2t + d) = 6t + 3d$$

Instructional Notes: A key learning for 6th grade is that students understand that substitution is a way to understand when two expressions are equivalent.

Multiplication and division in an expression represent a **single term**. ($5x$ is a term, $\frac{1}{2}x$ is a term)

Achievement Level Descriptors

Cluster: Apply and extend previous understandings of numbers to the system of rational numbers

Concepts and Procedures

Level 1: Students should be able to evaluate numerical expressions without exponents; write one- or two- step numerical expressions; and identify parts of an expression, using terms (e.g., coefficient, term, sum, product, difference, quotient, factor).

Level 2: Students should be able to evaluate numerical expressions with nonnegative integer exponents that do not need to be distributed across a set of parentheses. They should be able to apply and extend previous understandings of arithmetic to evaluate expressions with variables that do not contain exponents. They should also be able to write one- and two-step algebraic expressions that introduce a variable and identify equivalent expressions.

Level 3: Students should be able to write and evaluate numerical expressions with nonnegative integer exponents and expressions from formulas in real-world problems, and they should be able to apply and extend previous understandings of arithmetic to evaluate expressions with variables that include nonnegative integer exponents. They should be able to apply properties of operations to generate equivalent expressions.

Level 4: Students should be able to apply the understanding of the properties of operations and use the properties to show why two expressions are equivalent.