Unpacked South Dakota State Mathematics Standards

Purpose: *In order for students to have the best chance of success, standards, assessment, curriculum resources, and instruction must be aligned in focus, coherence, and rigor. Unpacked standards documents are intended to help align instruction to the focus, coherence, and rigor of the South Dakota State Mathematics Standards. The standards have been organized in clusters as they are not so much built from topics, but rather woven out of progressions. Not all content in a given grade is emphasized equally in the mathematics standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting standards will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.*

Domain: Geometry		**Grade Level: Geometry**
G.G.C.B Cluster: Find arc lengths and areas of sectors of circles This cluster explores the relationship between the length of an arc and the measure of a central angle. Learners develop a definition for the radian measure of an angle and apply radians to find the area of sectors.		
****This is an ADDITIONAL cluster.** *Students should spend the large majority of their time (65-85%) on the major work of the grade. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.* **G.G.C.B.5** Derive using similarity the length of the arc intercepted by an angle is proportional to the radius. **a.** Define the radian measure of the angle as the constant of proportionality **b.** Derive and apply the formula for the area of a sector.		
Aspects of Rigor: (Conceptual, Procedural, and/or Application)		
G.G.C.B.5a Derive using similarity the length of the arc intercepted by an angle is proportional to the radius. a. Define the radian measure of the angle as the constant of proportionality		
Conceptual Understanding	***Procedural Fluency***	***Application***
Understand how to define the radian measure of an angle using the ratio of the arc length and radius.	Find radian measures of angles given dimensions of parts of a circle including a combination of radius/diameter, central angle measure, or arc length.	
G.G.C.B.5b Derive using similarity the length of the arc intercepted by an angle is proportional to the radius. b. Derive and apply the formula for the area of a sector.		
Conceptual Understanding	***Procedural Fluency***	***Application***
Understand the steps for deriving the formula for the area of a sector.	Calculate the area of a sector given central angle measure and the length of a radius or diameter. Explain why the formula ($A = \frac{1}{2}r^2\theta$ with θ in radians) for the area of a sector works.	Solve contextual problems that ask for the area of a sector. For example, the area watered by a rotating sprinkler, or the area of sections of a flower garden.
Enacting the Mathematical Practices - Evidence of Students Engaging in the Practices		

1. **Make sense of problems and persevere in solving them.**
    - Perseverance and making sense of problems are fundamental to writing proofs.
    - Learners must be challenged to develop deep understanding of concepts through exploring tasks that require problem solving with a variety of tools and techniques.
2. **Reason abstractly and quantitatively.**
    - Use similarity to discover all circles are similar. Show similarity of circles with length of radii.
3. **Construct viable arguments and critique the reasoning of others.**
    - Write proofs and analyze proofs written by others.
    - Develop arguments to support conjectures and construct formal arguments when appropriate.
4. **Model with mathematics.**
    - There a multiple opportunities for modeling with circles, given their prevalence in the real-world.
    - Develop generalizations using modeling.
5. **Use appropriate tools strategically.**
    - Use compass and straightedge to construct figures.
    - Explore the parts of a circle and their relations using dynamic geometry software/tools.
6. **Attend to precision.**
    - Precision is of crucial importance in constructions, since even small errors in executing a construction may lead to results that don't work.
    - Use precise mathematical language to describe conjectures they develop.
7. **Look for and make use of structure.**
    - Many conjectures will be based on examination of underlying relationships within a problem, or determining a solution method that can be generalized into a theorem.
8. **Look for and express regularity in repeated reasoning.**
    - Look for patterns across a range of examples to better understand the behaviors of various parts of circles, triangles, and quadrilaterals.
    - Find various strategies for writing proofs and gain fluency selecting an effective argument strategy.

Vertical and Horizontal Coherence and Learning Progressions

Previous Learning Connections	***Current Learning Connections***	***Future Learning Connections***
In 7th grade, the formulas for the area and circumference of a circle are learned and then applied to solve problems. They give an informal derivation of the relationship between the circumference and area of a circle.	When calculating geometric probabilities, learners need to calculate the area of a sector.	In future courses, learners expand on their basic understanding of the radian measure of an angle. They apply radian measures when discovering relationships within the unit circle and while learning trigonometric relationships.

Vocabulary (key terms and definitions)

- arc length
- area of sector
- radian measure
- sector
- arc measure

Relevance, Explanations, and Examples:

Learners should not be simply applying a formula to convert angle measures in degrees to radians. The focus should be on using an arc length and radius measure to calculate the ratio.

When deriving the formula for the area of a sector, treat it as an application of the radian measure of an angle.

Before deriving the formula for the area of a sector, have learners find areas of sectors conceptually. This should naturally lead them to a derivation of the formula.

Achievement Level Descriptors	
Cluster: **Find arc lengths and areas of sectors of circles**	
Concepts and Procedures	***Level 1:*** Students should be able to base arguments on concrete referents such as objects, drawings, diagrams, and actions and identify obvious flawed arguments in familiar contexts.
	Level 2: Students should be able to find and identify the flaw in an argument by using examples or particular cases. Students should be able to break a familiar argument given in a highly scaffolded situation into cases to determine when the argument does or does not hold.
	Level 3: Students should be able to use stated assumptions, definitions, and previously established results and examples to test and support their reasoning or to identify, explain, and repair the flaw in an argument. Students should be able to break an argument into cases to determine when the argument does or does not hold.
	Level 4: Students should be able to use stated assumptions, definitions, and previously established results to support their reasoning or repair and explain the flaw in an argument. They should be able to construct a chain of logic to justify or refute a proposition or conjecture and to determine the conditions under which an argument does or does not apply.