**11th Grade Level 1 Understanding**

**Priority Cluster**

**Algebra**

* **Target D:** Interpret the structure of expressions.
  + Level 1 should be able to identify parts of an expression, such as terms, factors, coefficients, exponents, etc.
* **Target E:** Write expressions in equivalent forms to solve problems.
  + Level 1 students should be able to write a quadratic expression with integer coefficients and a leading coefficient of 1 in an equivalent form by factoring of exponents to expand a single variable (coefficient of 1) with a positive integer exponent into an equivalent form and vice versa, e.g. x3=xxx.
* **Target F:** Perform arithmetic operations on polynomials.
  + Level 1 students should be able to add, subtract and multiply single-variable polynomials of degree 2 or less.
* **Target G:** Create equations that describe numbers or relationships.
  + Level 1 students should be able to create and use one**-**step linear equations in one variable to model a familiar situation and to solve a familiar problem.
* **Target H:** Understand solving equations as a process of reasoning and explain the reasoning.
  + Level 1 students should be able to explain solution steps for solving one-step linear equations in one variable.
* **Target I**: Solve equations and inequalities in one variable.
  + Level 1 students should be able to solve one-step linear equations in one variable.
* **Target J:** Represent and solve equations and inequalities graphically.
  + Level 1 students should be able to represent a linear equation with an integer-valued slope in two variables graphically on a coordinate plane.

**Functions**

* **Target K:** Understand the concept of a function and use function notations.
  + Level 1 students should be able to distinguish between functions and nonfunctions. They should be able to state the domain and range given a graph.
* **Target L:** Interpret functions that arise in applications in terms of a context.
  + Level 1 students should be able to interpret linear functions in context, and given the key features of a linear graph, they should be able to identify the appropriate graph.
* **Target M:** Analyze functions using different representations.
  + Lvel1 students should be able to graph a linear function by hand or by using technology. They should be able to compare properties of two linear functions represented in different ways. They should be able to identify equivalent forms of linear functions.
* **Target N**: Build a function that models a relationship between two quantities.
  + Level1 students should be able to identify an explicit or recursive function and determine the steps for calculations from a contest requiring up to two steps. They should be able to add and subtract two linear functions.

**Statistics and Probability**

* **Target P:** summarize, represent, and interpret data on a single count or measurement variable.
  + Level 1 students should be able to describe a data set in terms of venter and spread and represent data graphically.

**Supporting Cluster**

**Quantities**

* **Target C:** Reason quantitatively and use units to solve problems.
  + Level 1 students should be able to choose the units in a formula, correctly scale a graph with unit increments, and identify a quantity from a graph with a scale in unit increments of a specified measurement.

**Number and Quantity**

* **Target A:** Extend the properties of exponents to rational exponents.
  + Level 1 students should be able to rewrite expressions with rational exponents of the form (1/n) to radical form and vice versa.
* **Target B**: use properties of rational and irrational numbers
  + Level 1 students should be able to identify the difference between a rational and an irrational number.

**Similarity, Right Triangles, and Trigonometry**

* **Target O**: define trigonometric rations and solve problems involving right triangles.
  + Level1 students should be able to identify trigonometric ratios and use the Pythagorean Theorem to solve for the missing side in a right triangle in familiar real-world or mathematical contexts with scaffolding.

**11th Grade Level 2 Understanding**

**Priority Cluster**

**Algebra**

**-Threshold:** The student who just enters Level 2 should be able to:

* + Use linear equations in one and two variables and inequalities in one variable to model a familiar situation and to solve a familiar problem.
  + Explain solution steps for solving linear equations and solve a simple radical equation.
  + Use properties of exponents to expand a single variable (coefficient of 1) repeated up to two times with a nonnegative integer exponent into an equivalent form and vice versa, e.g. e2x3=xxxxx=x2=3
  + Solve one-step linear equations and inequalities in one variable and understand the solution steps as a process of reasoning.
  + Represent linear equations and quadratic equations with integer coefficients in one and two variables graphically on a coordinate plane.
  + Recognize equivalent forms of linear expressions and write a quadratic expression with integer=leading coefficients in an equivalent form by factoring.
  + Add multi-variable polynomials made up of monomials of degree 2 or less.
  + Graph and estimate the solution of systems of linear equations.
* **Target D**: interpret the structure of expressions.
  + Level 2 students should be able to interpret parts of an expression such as terms, factors, coefficients, exponents, etc., and interpret simple compound expressions by viewing one or more of their parts as a single entity. They should also be able to recognize equivalent forms of linear expressions.
* **Target E**: Write expressions in eq1uivalent forms to solve problems.
  + Level 2 students should be able to write a quadratic expression with integer coefficients in an equivalent form by factoring or by completing the square. They should be able to use properties of exponents to expand a repeated single variables (coefficient of 1) with a nonnegative integer exponent into an equivalent form and vice versa, e.g. x0x2x3=xxxxxx=x2=3.
* **Target F**: Perform arithmetic operations on polynomials.
  + Level 2 students should be able to add, subtract, and multiply multi-variable polynomials made up of monomials of degree2 or less. They should understand that polynomials are closed under addition
* **Target G**: create equations that describe numbers of relationships.
  + Level 2 students should be able to create and use quadratic equations, linear equations, and linear inequalities in one and two variables to model a familiar situation and to solve a familiar problem. They should be able to graph a linear or a quadratic equation in two variables and be able to rearrange a familiar formula or an unfamiliar linear formula in one or two variables for a particular given quantity.
* **Target H:** understand solving equations as a process of reasoning and explain the reasoning
  + Level 2 students should be able to look for and make use of structure to solve simple radical equations and simple rational equations in one variable in which the variable term is in the numerator and should understand the solution steps as a process of reasoning. They should be able to understand and explain solution steps for solving linear equations in one variable as a process of reasoning.
* **Target I**: solve equations and inequalities in one variable.
  + Lvel2 students should be able to solve one-step linear inequalities and quadratic equations with integer coefficients in one and two variables graphically on a coordinate plane and should understand that the plotted line or curve represents the solution set to an equation. They should be able to graph and estimate the solution of systems of linear equations.

**Functions**

**-Threshold:** the student who just enters level 2 should be able to:

* Understand the concept of a function in order to distinguish a relation as a function or not a function.
* Interpret quadratic functions in context and given the key features of a graph, the student should be able to identify the appropriate graph.
* Graph quadratic functions by hand or by using technology.
* Identify properties of two linear or two quadratic functions.
* Understand equivalent forms of linear and quadratic functions.
* Build an explicit function to describe or model a relationship between two quantities.
* Add, subtract, and multiply linear functions.
* **Target K:** understand the concept of a function and use function notations.
  + Level 2 students should understand the concept of a function in order to distinguish a relation as a function or not a function. They should be able to identify domain and range of a function given a graph of a quadratic, linear, cubic, or absolute function, and they should understand that the graph of a function *f(x)* is the graph of the equation *y=f(x).*
* **Target L:** interpret functions that arise in applications in terms of a context.
  + Level 2 students should be able to interpret quadratic and other polynomials function in two variables in context of the situation, and given the key features of a graph of a polynomial function, they should be able to identify the appropriate graph. They should be able to specify the average rate of change from an equation of a linear function and approximate it from a graph of a linear function.
* **Target M:** Analyze functions using different representations.
  + Level 2 students should be able to graph linear and quadratic functions by hand; graph square root, cube root, piecewise-defined, polynomial, exponential, and logarithmic functions by hand or by using technology compare properties of two quadratic or two other functions of the same type, i.e., linear to linear, represented in different ways; and understand equivalent forms of linear and quadratic functions. They should be able to compare properties of two trigonometric functions represented in the same way.
* **Target N:** build a function that models a relationship between two quantities.
  + Level 2 students should be able to build an explicit or recursive function to describe or model a relationship between two quantities and determine the steps for calculation from a context. They should be able to add, subtract, and multiply linear and quadratic functions.

**Statistics and probability**

**-Threshold:** the student who just enters level 2 should be able to:

* Describe the differences in shape, center, and spread of two or more different data sets representing familiar contexts.
* **Target P:** summarize, represent, and interpret data on a single count or measurement variable.
* Level 2 students should be able to describe and use appropriate statistics to interpret and explain differences in shape, center, and spread of two or more different data sets including box plots, histograms, or dot plots, representing familiar contexts they should be able to identify the mean and the median and select the appropriate one for representing the center of the data for data sets.

**Supporting Cluster**

**Quantities**

**-Threshold:** the student who just enters level 2 should be able to:

* Choose and interpret the correct units in a formula given in a familiar context including making measurement conversions between simple units.
* Target C: Reason quantitatively and use units to solve problems.
* Level 2 students should be able to reason quantitatively to choose and interpret the units in a formula given in a familiar context, including making measurement conversions between simple units and identifying a quantity from a graph with the scale in increments of various sizes. They should be able to use units to guide the solution of a familiar multi-step problem with scaffolding.

**Number and Quantity**

**-Threshold:** the student who just enters level 2 should be able to:

* + Extend the properties of integer exponents to multiply expressions with rational exponents that have common denominators.
  + Perform operations on rational numbers and familiar irrational numbers.
  + Understand that rational numbers are closed under addition and multiplication
* **Target A:** extend the properties of exponents to rational exponents.
  + Level 2 students should be able to look for and use structure to extend the properties of integer exponents to multiply and divide expressions with rational exponents that have common denominators.
* **Target B:** Use properties of rational and irrational numbers.
  + Level 3 students should be able to perform operations n rational and irrational numbers and should be able to look for and use repeated reasoning to understand that the rational numbers are closed under addition and multiplication.

**Similarity, Right Triangles, and Trigonometry**

**-Threshold:** The student who just enters level 2 should be able to:

* + Use the Pythagorean Theorem in unfamiliar problems to solve for the missing side in a righttriangle with some scaffolding.
* **Target O:** define trigonometric ratios and solve problems involving right triangles.
  + Level 2 students should be able to define trigonometric ratios and should know the relationship between the sine and cosine of complementary angles. They should be able to use the Pythagorean Theorem in unfamiliar problems and trigonometric ratios in familiar problems to solve for the missing side in a right triangle with some scaffolding.

**11th Grade Level 3 Understanding**

**Priority Cluster**

**Algebra**

**-Threshold:** the student who just enters level 3 should be able to:

* + Create and use quadratic inequalities in two variables to model a situation and to solve a problem.
  + Write a quadratic expression in one variable with rational coefficients in an equivalent form by factoring, identify its zeros, and explain the solution steps as a process of reasoning.
  + Use properties of exponents to write equivalent forms of exponential functions with one or more variables with integer coefficients with nonnegative integer exponents involving operations of addition, subtraction and multiplication without requiring distribution of an exponent across parentheses.
  + Solve a quadratic equation with integer roots in standard form.
  + Represent polynomial and exponential functions graphically and estimate the solution of systems of equations displayed graphically.
  + Understand that the plotted line, curve or region represents the solution set to an equation or inequality.
  + Add and subtract multi-variable polynomials of any degree and understand that polynomials are closed under subtraction.
* **Target D:** interpret the structure of expressions.
  + Level 3 students should be able to recognize equivalent forms of expressions and use the structure of an expression to identify ways to rewrite it. They should be able to interpret complicated expressions by viewing one or more of their parts as a single entity.
* **Target E:** Write expressions in equivalent forms to solve problems.
  + Level 3 students should be able to write a quadratic expression with rational coefficients in an equivalent form by factoring and by completing the square. They should be able to identify and use the zeros to solve of explain familiar problems and they should be able to use properties of exponents to write equivalent forms of exponential functions with one or more variables, integer coefficients, and nonnegative rational exponents involving operations of addiction, subtraction, and multiplication, including distributing an exponent across terms within parentheses.
* **Target F:** perform arithmetic operations on polynomials.
  + Level 3 student should be able to add, subtract, and multiply multi-variable polynomials of any degree and understand that polynomials are closed under subtraction and multiplication.
* **Target G**: Create equations that describe numbers or relationships.
  + Level 3 students should be able to create and use linear, quadratic, and rational equations and inequalities and exponential equations with an integer base and a polynomial exponent in multiple variables to model an unfamiliar situations and solve an equation in two variables and be able to rearrange a linear, a quadratic, an absolute, a rational or a cubic multi-variable formula for a particular given quantity.
* **Target H:** understand solving equations as a process of reasoning and explain the reasoning.
  + Level 3 students should be able to look for and make use of structure to solve simple radical and rational equations in one variable presented in various forms. They should be able to understand and explain solution steps for solving quadratic, radical, and rational equations in one variable as a process of reasoning.
* **Target I:** Solve equations and inequalities in one variable.
  + Level 3 students should be able to solve multi=step linear equations and inequalities and quadratic equations in one variable with real roots.
* **Target J**: represent and solve equations and inequalities graphically.
  + Level 3 students should be able to represent polynomial, rational, absolute value, exponential, and logarithmic functions graphically. They should be able to graph and estimate the solution of systems of equations and systems of linear inequalities. They should understand that the plotted line, curve, or region represents the solution set to an equation or inequality

**Functions**

**-Threshold:** the student who just enters level 3 should be able to:

* + Identify the domain and range of linear, quadratic, and exponential functions presented in any form. Use function notation to evaluate a function for numerical or monomial inputs.
  + Appropriately graph and interpret key features of linear, quadratic, and exponential functions in familiar or scaffolded contexts and specify the average rate of change of a function on a given domain from its equation or approximate the average rate of change of a function from its graph.
  + Graph linear, quadratic, logarithmic, and exponential functions by hand and by using technology.
  + Analyze and compare properties of a linear function to properties of another function of any type. Build a recursive function to describe or model a relationship between two quantities.
  + Divide linear functions.
* **Target K:** Understand the concept of a function and use function notations.
  + Level 3 students should be able to use function notation to evaluate a function given in function notation for a particular input. They should be able to identify the domain and range for any given function presented in any form, e.g., as a graph, a verbal description, or a sequence.
* **Target L:** interpret functions that arise in applications in terms of a context.
  + Level 3 students should be able to graph various types of function and interpret and relate key features, including range and domain, in familiar or scaffolded contexts. They should be able to specify the average rate of change of a function on a given domain from its equation or approximate the average rate of change of a function from its graph.
* **Target M:** analyze functions using different representations.
  + Level 3 students should be able to analyze and compare properties of two functions of different types represented in different ways and understand equivalent forms of functions. They should be able to graph trigonometric functions by hand and by using technology.
* **Target N:** build a function that models a relationship between two quantities.
  + Level 3 students should be able to translate between explicit and recursive forms of a function. They should be able to add, subtract, multiply, and divide functions.

**Statistics and Probability**

**-Threshold:** the student who just enters level 3 should be able to:

* + Select the appropriate choice of spread as interquartile range or standard deviation based on the selection of the measure of center.
* **Target P:** summarize, represent, and interpret data on a single count or measurement variable.
  + Level 3 students should be able to use appropriate statistics to interpret, explain, and summarize differences in shape, center, and spread of two or more different data sets of varying complexity and levels of familiarity, including the effect of outliers. They should be able to select the appropriate choice of spread as interquartile range or standard deviation based on the selection of center and use the standard deviation of a data set to fit to a normal distribution.

**Supporting Cluster**

**Quantities**

* **Threshold:** the student who just enters level 3 should be able to:
  + Reason quantitatively to choose and interpret the units in a formula given in an unfamiliar context including making compound measurement conversions.
  + Define appropriate quantities or measurements in familiar contexts with some scaffolding to construct a model.
  + Choose the scale and origin of a graph or data display
* **Target C:** reason quantitatively and use unites to solve problems.
  + Level 3 students should be able to reason quantitatively to choose and interpret the units in a formula given in an unfamiliar context including making measurement conversions between compound units, and to define appropriate quantities or measurements in familiar contexts with some scaffolding to construct a model. They should be able to identify appropriate levels of measurement precision in context and to choose and interpret the scale and origin of a graph or data display. They should be able to use units to guide the solution of an unfamiliar multi-step problem without scaffolding.

**Number and Quantity**

**-Threshold**: the student who just enters level 3 should be able to:

* + Apply all laws of exponents on expressions with exponents that have common denominators.
  + Rewrite expressions with rational exponents of the form (m/n) to radical form and vice versa.
  + Use repeated reasoning to recognize that the sums and products of a rational number and a nonzero irrational number are irrational.
* **Target A:** extend the properties of exponents to rational exponents.
  + Level 3 students should be able to rewrite expressions with rational exponents of the form (m/n) to radical form, and vice versa, and look for and use structure to extend the properties of integer exponents to all laws of exponents on radical expressions and expressions with rational exponents.
* **Target B:** use properties of rational and irrational numbers.
  + Level 3 students should be able to look for and use repeated reasoning to understand and explain that the sum and product of a rational number and a nonzero irrational number are irrational.

**Similarity, Right Triangles, and Trigonometry**

**-Threshold:** the student who just enters level 3 should be able to:

* + Use trigonometric ratios and the sine and cosine of complementary angles to find missing angles or sides of a given right triangle with minimal scaffolding.
* **Target O:** Define trigonometric ratios and solve problems involving right triangles.
  + Level 3 students should be able to use the Pythagorean Theorem, trigonometric ratios and the sine and cosine of complementary angles to solve unfamiliar problems with minimal scaffolding involving right triangles, finding the missing side or missing angle of a right triangle.

**11th Grade Level 4 Understanding**

**Priority Cluster**

**Algebra**

**-Threshold:** the student who just enters level 4 should be able to:

* + Choose an appropriate equivalent form of an expression in order to reveal a property of interest when solving problems.
  + Solve a formula for any variable in the formula.
  + Provide an example that would lead to an extraneous solution when solving linear, quadratic, radical, and rational equations.
  + Use a variety of methods such as factoring, completing the square, quadratic formula, etc., to solve equations and to find minimum and maximum values of quadratic equations
* **Target D:** interpret the structure of expressions.
  + Level 4 students should be able to look for and use structure and repeated reasoning to make generalizations about the possible equivalent forms expressions can have e.g., a quadratic expression can always be represented as the product of two factors containing its roots.
* **Target E:** write expressions in equivalent forms to solve problems.
  + Level 4 students should be able to find the maximum or minimum values of a quadratic function. They should be able to choose an appropriate equivalent form of an expression in order to reveal a property of interest when solving problems.
* **Target F:** perform arithmetic operations on polynomials.
  + Level 4 students should understand and be able to explain that polynomials form a system analogous to the integers.
* **Target G:** create equations that describe numbers or relationships.
  + Level 4 students should be able to rearrange polynomial, logarithmic, exponential, or trigonometric formulas with one or more variables to highlight a quantity of interest and be able to analyze in context to determine which quantity is of interest.
* **Target H:** understand solving equations as a process of reasoning and explain the reasoning.
  + Level 4 students should be able to give examples showing how extraneous solutions may arise and why they arise when solving linear, quadratic, radical, and rational equations.
* **Target I:** solve equations and inequalities in one variable.
  + Level 4 students should be able to solve quadratic equations in one variable with complex roots.
* **Target J:** Represent and solve equations and inequalities graphically.
  + Level 4 students should be able to explain why the x-coordinates of the points where *f(x)* and *g(x)* intersect compose the solution to *f(x)=g(x)*

**Functions**

**-Threshold:** the student who just enters Level 4 should be able to:

* + Find the input of a function when given the function in function notation and the output, or find the output when given the input.
  + Describe complex features such as holes, symmetries, and end behavior of the graph of a function.
  + Graph functions both by hand and by using technology.
* **Target K:** understand the concept of a function and use function notations.
  + Level 4 students should be able to find the input for a given output when given in function notation.
* **Target L:** interpret functions that arise in applications in terms of context.
  + Level 4 students should be able to interpret complex key features such as holes, symmetries, and end behavior of graphs and functions in unfamiliar problems or contexts.
* **Target M:** Analyze functions using different representations.
  + Level 4 students should be able to graph a variety of functions including linear quadratic square root cube root piecewise-defined polynomial exponential logarithmic and trigonometric, by hand and by using technology. They should be able to analyze and explain relationships between various types of functions and the behaviors of the functions and be able to determine which equivalent form is most appropriate for a given task.
* **Target N:** build a function that models a relationship between two quantities.
  + Level 4 students should be able to determine when it is appropriate to combine functions using arithmetic operations in context

**Statistics and Probability**

**-Threshold:** the student who just enters level 4 should be able to:

* + Interpret data to explain why a data value is an outlier.
* **Target P:** summarize, represent, and interpret data on a single count or measurement variable.
  + Level 4 students should be able to interpret data to explain why a data value is an outlier and interpret and explain differences in the approximate areas under the normal curve of two or more data sets.

**Supporting Cluster**

**Quantities**

**-Threshold:** the student who just enters Level 4 should be able to:

* + Define appropriate quantities or measurements in unfamiliar contexts with some scaffolding to construct a model.
* **Target C:** reason quantitatively and use unites to solve problems.
  + Level 4 students should be able to define appropriate quantities or measurements in unfamiliar contexts with little to no scaffolding to construct a model.

**Number and Quantity**

**-Threshold:** the student who just enters level 4 should be able to:

* + Explain the relationship between properties of integer exponents and properties of rational exponents.
* **Target A:** Extend the properties of exponents to rational exponents.
  + Level 4 students should be able to identify the exponent property used when rewriting expressions and recognize when laws of exponents cannot be used to rewrite an expression.
* **Target B:** use properties of rational and irrational numbers.
  + Level 4 students should be able to provide a specific example given a generalization statement, such as the sum of a rational number and an irrational number is irrational.

**Similarity, Right Triangles, and Trigonometry**

**-Threshold:** the student who just enters level 4 should be able to:

* + Solve right triangle problems with multiple stages and in compound figures without scaffolding.
* **Target O:** define trigonometric ratios and solve problems involving right triangles.
  + Level 4 students should be able to solve unfamiliar, complex, or multi-step problems without scaffolding involving right triangles.