

South Dakota Performance Level Descriptors—Math

	Level 1	Level 2	Level 3	Level 4
	Students in Level 1 can demonstrate some of the following skills:	Students in Level 2 can likely demonstrate the skills in Level 1 and can demonstrate some of the following skills:	Students in Level 3 can likely demonstrate the skills in Levels 1 and 2 and can demonstrate some of the following skills:	Students in Level 4 can likely demonstrate the skills in Levels 1, 2, and 3 and can demonstrate some of the following skills:
Number & Quantity	• Locate rational numbers on the number line • Define number systems, including real, imaginary, and complex • Understand the meaning of whole number exponents and multiply like terms such as $2b^3$ and $5b^5$	• Perform multiple numerical operations to solve simple arithmetic problems in real-world contexts • Convert units in rate problems when the conversion is given • Multiply and divide numbers written in scientific notation • Use set notation and logic to describe events • Evaluate numerical expressions involving simple square roots or rational exponents • Bound the square root of a whole number between two whole numbers	• Determine whether the sum or product of two real numbers is rational or irrational • Determine if a mathematical solution makes sense in context • Convert units in real-world contexts to solve problems when conversions are given • Evaluate numerical expressions involving more complex square roots or simple cube roots • Express radical expressions using rational exponents • Use properties of exponents to multiply, divide, and apply the power rule to monomials with integer exponents. Rewrite the expression in fractional form with whole number exponents • Solve for a given variable when exponents and radicals are involved • Use structure to add and subtract complex numbers in $a + bi$ form, where a and b are integers	• Explain why the sum or product of two real numbers is rational or irrational • Approximate irrational numbers to order a set of numbers • Convert units in rate problems to solve multistep or multiple-rate problems • Evaluate numerical expressions involving square roots and cube roots of fractions • Multiply monomials with the same base and rational exponents • Reason using number properties (odd, even, negative, positive, integer, rational, irrational, etc.) • Find complex solutions to quadratic equations with real coefficients • Use structure to multiply complex numbers in $a + bi$ form, where a and b are integers, given $i^2 = -1$

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Algebra	- Define algebraic objects, including terms, factors, variables, constants, coefficients, functions, equations, and polynomials - Build an equation of the form $y = ax + b$ from a simple real-world context where the equation is a close parallel to the description - Write expressions in one variable to represent a quantity - Find the value of a simple algebraic expression for whole number values of the variables and identify appropriate values of the variables from a simple context - Know that a solution for an equation means that the solution value makes the equation true and use that to find the value of a parameter in a simple equation family - Solve or verify the solution to a linear equation in one variable where the variable is in a single term and the solution is an integer	- Use proportional reasoning given a ratio or scale expressed as one whole number to another, and given one of the quantities that is a whole number multiple of its base in the ratio, find the other quantity - Convert equations from standard form to slope-intercept form to determine slope and y-intercept - Solve problems that could be modeled as $y = mx + b$ - Write equations in one variable to represent contexts and to solve problems - Rewrite algebraic expressions by applying a single algebraic property - Set up expressions, equations, or inequalities in one variable to represent real-world contexts and solve problems - Translate verbal expressions into algebraic expressions when the order of quantities and operations within the verbal description is the same as the order within the algebraic expression and including cases with multiple variables - Solve linear equations in one variable with integer coefficients and constants - Identify a variable pair that satisfies two linear conditions such as equations or inequalities (e.g., cost, quantity, profit) - Solve a system of two linear equations in two variables where one of the equations is in one variable - Solve quadratic equations of the form $ax^2 = b$ to find the positive solution	- Solve linear inequalities in one variable - Isolate a literal equation for one variable, where coefficients and constants are rational numbers and computations are simple - Evaluate a quadratic expression or an absolute value expression - Solve quadratic equations where solutions are real numbers using a variety of methods, including factoring to identify zeros, using the quadratic formula, and completing the square - Interpret linear, quadratic, and exponential expressions in terms of the context they represent - Rewrite algebraic expressions in equivalent forms by applying multiple algebraic properties. Use the new form to reveal properties of an expression - Add and subtract polynomials, including higher-order polynomials - Apply the distributive property when multiplying a constant by a binomial or a binomial by a binomial - Use the definition of a rational exponent as a radical to rewrite expressions and solve equations - Rewrite expressions involving radicals that include addition, subtraction, and multiplication - Write a linear equation in two variables given a context, table, or points - Solve systems of two linear equations in two variables	- Isolate a literal equation for one of the variables, where coefficients and constants are real numbers and computations include exponents or radicals - Compare different representations of the same information, including graphs, equations, and tables - Identify quadratic equations that have no real solution or that have a given complex number as a solution - Solve quadratic equations using a variety of methods, where solutions can be real numbers or complex numbers - Write equations in one variable, including absolute value, quadratic, and exponential, to represent real-world contexts and to solve multistep problems - Interpret structure to identify equivalent expressions - Multiply more complex polynomials - Use the structure of an expression to factor a polynomial - Find the zeros of a polynomial function that can be factored - Perform operations on polynomial and rational expressions - Simplify cube-root expressions - Solve rational and radical equations and eliminate extraneous solutions - Rewrite expressions involving radicals that include addition, subtraction, multiplication, and division to solve problems in a geometric or algebraic context

		Factor a quadratic equation with integer coefficients and where the factors have integer coefficients	- Graph the solution set to a linear inequality in two variables - Identify a variable pair that satisfies three linear conditions, such as a system of equations or inequalities	- Graph the solution set to a system of linear inequalities - Use a graph to solve a simple system consisting of a linear equation and a quadratic equation in two variables
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Functions	- Identify parts of an equation that are the same and that are different (for example, in $3^{(a+1)} = 3^4$) and recognize that making the different parts equal will provide a solution to the equation - Identify real zeros from a graph - Use closely parallel structure to accomplish tasks such as identifying the values of variables from a context and substituting into an expression when the context presents the values in the same order as in the expression	- Determine when a relation is a function - Find the value of a function using function notation - Find a linear function when given ordered pairs in a table or to represent a context - Find slope from an equation in standard form - Calculate and interpret the average rate of change of a function over an interval - Given the first few terms in an arithmetic or geometric sequence, find one of the next few terms - Graph linear functions - Identify key features (e.g., slope, intercepts, domain, or range) of a linear function given a graph, table, or equation	- Use function notation and evaluate functions representing a context - Associate constant rate of change with a linear graph - Understand and compare the growth or decay of a function as the input increases or decreases - Given the first few terms of an arithmetic or geometric sequence, write an explicit formula - Given two terms of a geometric sequence, write an explicit formula - Graph quadratic, exponential, step, and absolute value functions and perform transformations on the graphs of these functions - Interpret key features (e.g., min, max, domain, range, intercepts) of a quadratic graph, table, or equation - Write equations in two variables, including quadratic, exponential, absolute value, and piecewise equations, to represent real-world contexts and to solve multistep problems - Relate real-world situations to the domain and range of functions - Compare properties of two functions, including quadratic, exponential, absolute value, and piecewise, represented in different forms - Given a factor of a quadratic polynomial, find the other factor, where coefficients are integers - Recognize a common factor of a polynomial in one or two variables	- Build a function for a sequence and use it to find other terms - Given a recursive expression involving subscripts for the next term in a sequence, find the next term - Given a recursive formula for a sequence, find the subsequent terms. Build a function to represent a sequence. Find the sum of the terms in a finite or infinite series - Use sequences in real-world contexts to solve problems - Write equations in two variables, including polynomial, trigonometric, logarithmic, radical, and piecewise, to represent real-world contexts and to solve multistep problems - Connect a factorization of a higher-order polynomial with its graph - Interpret a linear or exponential model of a real-world situation - Graph polynomial, trigonometric, logarithmic, reciprocal, radical, rational, and piecewise functions. Perform transformations on the graphs of these functions - Identify key features of a polynomial and of trigonometric, exponential, logarithmic, reciprocal, radical, rational, and piecewise functions - Estimate solutions to equations or systems of equations or other quantifiable features by graphing - Find an expression for a composition of two functions and use the notation $f \circ g(x)$ - Build and evaluate inverse functions to solve real-world problems

			• Evaluate logarithmic functions using properties of exponents and logarithms • Evaluate composite functions for integer inputs • Find the inverse of a one-to-one function	• Find the inverse of a function by restricting the domain of the function • Use the relationship between exponents and logarithms to solve logarithmic equations • Verify by composition that one function is an inverse of another • Evaluate trigonometric functions for all real values • Model real-world situations using trigonometric functions. Solve trigonometric equations • Use the Pythagorean identity to calculate trigonometric ratios
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Geometry	- Define geometric objects such as points, planes, rays, line segments, angles, parallel and perpendicular lines, and circles - Identify corresponding sides of congruent triangles using the structure of congruence notation - Perform a single transformation on points in the coordinate plane - Determine the length, width, or area of a rectangle when given the other two quantities - Use geometric formulas to find perimeter, area, or volume when given all needed quantities	- Perform a single rigid transformation on a point or a figure in the coordinate plane - Identify the reflection of a point across an axis - Find horizontal and vertical distances in the coordinate plane to solve problems - Given distances from two endpoints to a point in between, find the distance from one endpoint to the midpoint on the number line - Use geometric formulas to find the missing value given perimeter or area - Determine if a triangle is a right triangle given the three side lengths. Given two sides of a right triangle, find the third side	- Identify the effect on the graph of a function when an integer is added to the function to cause a translation - Perform transformations on figures and graphs. Transformations include a single dilation, a single rotation, or a series of translations. Perform a reflection of a point over a line - Describe the rotation that takes a figure onto itself - Apply standard congruence relationships based on translation, rotation, and reflection, such as for base angles of isosceles triangles, angle relationships formed when parallel lines are crossed by a transversal, and vertical angles - Calculate distances on the coordinate plane to solve advanced problems - Find the midpoint given two points on the coordinate plane - Find the perimeter or area of a polygon given its vertices in the coordinate plane - Identify patterns and relationships in figures such as lines, triangles, quadrilaterals, and circles - Add and subtract areas to find the area of a composite shape - Capture geometric reasoning through algebraic relationships - Find side lengths for a triangle or quadrilateral with a given perimeter and with relationships between side lengths	- Extend application of transformations in the coordinate plane of figures or graphs to include reflections across a line and a series of multiple transformations - For a point, figure, or graph, identify coordinates, the figure, or the graph after multiple translations - Find the point on a line segment between two given points that divides the line segment in a given ratio - Calculate distances on the number line to solve advanced problems - Use relationships between slopes and intercepts to build linear functions with specified properties (e.g., parallel and perpendicular lines) - Prove angle relationships such as for base angles of isosceles triangles, angle relationships formed when parallel lines are crossed by a transversal, and vertical angles and use them to solve problems - Apply the triangle inequality theorem to find the range of possible lengths for the third side given two side lengths - Prove theorems about triangles and quadrilaterals and use them to solve problems - Recognize and apply similarity in conjunction with other relationships between lengths - Prove and apply theorems related to similarity of triangles - Compute the area and circumference of circles in a coordinate plane after identifying necessary information

			● Find angle measures using the relationship between interior angles in a triangle and the relationship between adjacent angles ● Find the area of a triangle when given the length of two sides and the measure of the included angle ● Use the relationship between similar figures to find side lengths and angle measures of similar triangles. Apply similarity to solve applications involving scale drawings ● Find the length of a midsegment in a triangle ● Use similar circles to find missing lengths ● Calculate arc lengths and areas of sectors in circles ● Apply the relationships among central angles, inscribed angles, radii, and chords to solve problems involving circles ● Make formal geometric constructions: Copy segments and angles; bisect segments and angles; construct perpendicular lines, parallel lines, equilateral triangles, squares, and a regular hexagon inscribed in a circle ● Find the equation of a circle when given points on a horizontal or vertical diameter by using midpoint and distance formulas ● Identify the center and radius of a circle given an equation ● Find the length, width, height/depth, or volume of a rectangular prism when given the other three quantities ● Identify the three-dimensional figure formed by rotating a two-dimensional figure ● Convert between degrees and radians ● Use the Pythagorean theorem to find the length of the third side of a right triangle and	● Find the equation of a circle given points on any diameter by using midpoint and distance formulas ● Recognize the relationship between a tangent and a circle ● Solve problems involving the relationship between the area of a sector, the central angle measure, and the radius of a circle ● Use chord-radius relationships in a circle ● Extend the chord-radius relationship to define and apply radian measure ● Identify and describe relationships among inscribed and circumscribed angles and tangents to circles ● Identify the properties needed to define and construct figures such as lines, triangles, quadrilaterals, and circles ● Using geometric constructions, define functions such as rotations, reflections, and translations ● Add and subtract volumes to find the volume of a composite shape or shape with portions removed ● Use geometric formulas to find area, surface area, or volume when values are not given directly for all needed variables ● Find perimeters, areas, and volumes when given complex relationships between dimensions ● Identify the shapes of two-dimensional cross sections of three-dimensional objects ● Prove the Pythagorean theorem and its converse ● Use the converse of the Pythagorean theorem to know whether angle measures are more than, less than, or equal to 90°
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			then express a trigonometric ratio using the third side length of a right triangle • Relate angle measures to the ratio of side lengths of a right triangle through sine, cosine, or tangent, or through an inverse function. Evaluate trigonometric functions for some real values • Use trigonometric ratios to find missing side lengths and angles of right triangles. Apply the constant ratios of the sides in special right triangles	• Strategically employ the Pythagorean theorem to find lengths when solving real-world problems • Use trigonometric ratios and the Pythagorean theorem to find all the missing side lengths and angles of right triangles
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Statistics & Probability	- Find a median when given an odd number of data points - Calculate the average of a list of numbers - Find the probability of a simple event happening or not happening when the sample space is clear - Interpret a value from one region of a Venn diagram	- Find a missing data value when given the average/mean and all but one data value - Distinguish between correlation and causation - Represent data for two variables on a scatterplot or a two-way table - Find a conditional probability when it is straightforward to apply the condition to the situation and then find a simple probability within the new situation - Interpret a frequency from a Venn diagram in terms of “not” - Determine if the results of a probability experiment are consistent with the expected results - Interpret data points in a scatterplot representing a context and describe how the variables are related	- Find median from a frequency table or a stem-and-leaf plot - Use statistics to compare center and spread of two or more datasets, including mean, median, interquartile range, and standard deviation - Identify possible trends and associations between two categorical variables by using two-way frequency tables, segmented bar graphs, and relative frequencies - Interpret data in a histogram, box plot, or dot plot - Interpret differences in shape, center, and spread of datasets and understand the effect outliers have on measures of center and spread - Recognize that the greater the standard deviation, the greater the spread from the mean - Understand how changes in a dataset affect the measures of center and spread - Identify whether a survey is random or nonrandom in a real-world context - Analyze and describe a dataset with a normal distribution, including estimating population percentages - Find the probability of a compound event formed from multiple independent events happening, such as a coin toss, roll of dice, or single drawing - Compute a probability when the sample space is not obvious, including finding conditional probabilities	- Use standard deviation to interpret information given a statistical model - Given a verbal description of the objects in a set, the subsets within the set, and the overlaps, find the number of objects in a particular subset - Analyze bivariate quantitative and categorical data to make conclusions and assess risk - Understand the choices to be made when creating statistical studies and how those choices can lead to biases and flaws in the study - Estimate a population mean or proportion by interpreting data from a survey - Find the probability of a compound event with frequencies given in a two-way table, defined by a certain sum of outcome values from independent events and using random draws with or without replacement - Identify events in real-world contexts as independent or dependent - Find a geometric probability where the sample space and the event can be represented by two-dimensional shapes - Find the probability that event A or event B will occur given they are mutually exclusive events and given the probability of event A and event B - Use probabilities, including expected value, to make and analyze decisions - Interpret data using a model of a linear function. Make predictions based on a linear model

			● Given the probability of each simple event, where events are defined by a single trait (e.g., color), find the probability of a compound event described with “and,” “or,” and “not” ● Model data with linear functions and interpret the slope and y-intercept of the line of best fit in terms of the context ● Interpret the correlation coefficient for a model	● Model data with nonlinear functions
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