# Unpacked South Dakota State Mathematics Standards

***Purpose:*** *In order for students to have the best chance of success, standards, assessment, curriculum resources, and instruction must be aligned in focus, coherence, and rigor. Unpacked standards documents are intended to help align instruction to the focus, coherence, and rigor of the South Dakota State Mathematics Standards. The standards have been organized in clusters as they are not so much built from topics, but rather woven out of progressions. Not all content in a given grade is emphasized equally in the mathematics standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting standards will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.*

| **Domain: Number and Quantity** | | **Grade Level: 12** |
|---|---|---|
| **HS4.N.VM.C Cluster: Perform operations on matrices and use matrices in applications.**<br>Students learn to use matrices to represent complex situations and how to manipulate matrices. | | |
| **This is a MAJOR cluster for a Discrete Mathematics course.**<br>**This is an ADDITIONAL cluster for a Precalculus course.**<br>*Students should spend the large majority of their time (65-85%) on the major work of the grade. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.*<br><br>**N.VM.6** Use matrices to represent and manipulate data.<br>**N.VM.7** Multiply matrices by scalars to produce new matrices.<br>**N.VM.8** Add, subtract, and multiply matrices of appropriate dimensions.<br>**N.VM.9** Understand that, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.<br>**N.VM.10** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. Discover that the determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.<br>**N.VM.11** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.<br>**N.VM.12** Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. | | |
| **Aspects of Rigor for Student Learning:** (Conceptual, Procedural, and/or Application) | | |
| **N.VM.6** Use matrices to represent and manipulate data. | | |
| ***Conceptual Understanding*** | ***Procedural Fluency*** | ***Application*** |
| Students understand that a matrix can be used to represent multivariate data.<br><br>Students understand that they can add, subtract, or multiply matrices with physical data to develop new matrices with additional meaning. | Students can build a matrix with data. | Students can solve problems by creating matrices from physical data and manipulating those matrices to create solution data sets. |
| **N.VM.7** Multiply matrices by scalars to produce new matrices. | | |
| ***Conceptual Understanding*** | ***Procedural Fluency*** | ***Application*** |

| | | |
|---|---|---|
| | Students can multiply matrices by scalars to produce new matrices. | |
| **N.VM.8** Add, subtract, and multiply matrices of appropriate dimensions. | | |
| ***Conceptual Understanding*** | ***Procedural Fluency*** | ***Application*** |
| Students understand that the dimensions of the matrix is important. | Students can add, subtract, and multiply matrices. | |
| **N.VM.9** Understand that, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. | | |
| ***Conceptual Understanding*** | ***Procedural Fluency*** | ***Application*** |
| Students understand which properties from real numbers can be applied to matrices. | | |
| **N.VM.10** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. Discover that the determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. | | |
| ***Conceptual Understanding*** | ***Procedural Fluency*** | ***Application*** |
| Students can relate the zero and identity matrix to real numbers 0 and 1. | Students can find the determinant of a square matrix. | |
| **N.VM.11** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. | | |
| ***Conceptual Understanding*** | ***Procedural Fluency*** | ***Application*** |
| Students understand that multiplying a matrix and a vector together transforms the vector. | Students can perform transformations by multiplying a matrix by a vector. | |

**N.VM.12** Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

| ***Conceptual Understanding*** | ***Procedural Fluency*** | ***Application*** |
|---|---|---|
| Students understand that the determinant of a transformation matrix acts as a scale factor for the increase or decrease in area after the transformation. | Students can compute the area of a parallelogram using the determinant. | |

**Enacting the Mathematical Practices - Evidence of Students Engaging in the Practices**

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
    - Students need to think abstractly to relate matrices to physical shapes and transformations of those shapes.
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
    - Students can use matrices to use mathematics to model real world situations.
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
    - Matrices provide structure to allow for display and manipulation of data.
8. **Look for and express regularity in repeated reasoning.**

**Vertical and Horizontal Coherence and Learning Progressions**

| ***Previous Learning Connections*** | ***Current Learning Connections*** | ***Future Learning Connections*** |
|---|---|---|
| In elementary school, students were introduced to rectangular arrays of data as an introduction to multiplication.<br><br>In Geometry, students explored transformations.<br><br>In prior courses, students have explored the mathematical properties of real numbers. | Students work with vectors and matrices.<br>Students should see connections between their work with vectors and matrices to geometry (transformations) and algebra (properties). | Students will use vectors and matrices in Linear Algebra, or in a Discrete Math course. Some work with matrices can be found in a College Algebra course. |

***Vocabulary (key terms and definitions)***

- Zero Matrix
- Identity Matrix
- Determinant
- Scalar

***Relevance, Explanations, and Examples:***

One example of using the determinant for finding area is:

If we have a parallelogram with vertices at *O* (0, 0), *P* (2, 4), *Q* (3, 9), and *R*(1, 5), we can make the matrix:

$$\begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$$

Its determinant is 2 × 5 – 1 × 4 = 10 – 4 = 6. And the absolute value of 6 is also 6. So the area of parallelogram *OPQR* is 6 units$^2$