Unpacked South Dakota State Mathematics Standards

Purpose: *In order for students to have the best chance of success, standards, assessment, curriculum resources, and instruction must be aligned in focus, coherence, and rigor. Unpacked standards documents are intended to help align instruction to the focus, coherence, and rigor of the South Dakota State Mathematics Standards. The standards have been organized in clusters as they are not so much built from topics, but rather woven out of progressions. Not all content in a given grade is emphasized equally in the mathematics standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting standards will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.*

Domain: Statistics and Probability	**Grade Level: Geometry**

G.S.CP.A Cluster: Understand independence and conditional probability and use them to interpret data
A probability model may consist of a list or description of possible outcomes (the sample space) each of which is assigned a certain probability. Probability rules can be developed and understood through the use of the sample space. When events are independent, the outcome of the first event does not change the sample space for subsequent events. In dependent events, knowing one event has occurred affects the likelihood of another event occurring. Use of two-way frequency tables helps learners develop conceptual understanding of conditional probability. The use of tables, symbols, and real-world scenarios are emphasized. Learners consider the context of situations as they build mathematical models, interpret events, and explain results in terms of a probability model.

****This is an ADDITIONAL cluster.** *Students should spend the large majority of their time (65-85%) on the major work of the grade. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.*

G.S.CP.A.1 Describe events as subsets of a sample space or as unions, intersections, or complements of other events.

G.S.CP.A.2 Determine whether two events A and B are independent.

G.S.CP.A.3 Determine conditional probabilities and interpret independence by analyzing conditional probability.

G.S.CP.A.4 Construct and interpret two-way frequency tables of data. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Aspects of Rigor: (Conceptual, Procedural, and/or Application)

G.S.CP.A.1 Describe events as subsets of a sample space or as unions, intersections, or complements of other events

Conceptual Understanding	***Procedural Fluency***	***Application***
Event outcomes can be represented using sample spaces. A sample space may have multiple subsets. A compound event is one in which there is more than one possible outcome.	Use tree diagrams, organized lists, tables, and/or Venn diagrams to represent sample spaces. Determine unions of sample spaces. Determine intersections of sample	

Two or more sample spaces can be combined to form a union of the spaces. Two or more sample spaces can have the same subset creating an intersection. Complements of an event are the remainder of the sample space minus the event. Set notation can be used to represent contextual problems.	spaces. Determine complements of sample sets. Represent unions, intersections, and complements using set notation.	
G.S.CP.A.2 Determine whether two events A and B are independent.		
Conceptual Understanding	***Procedural Fluency***	***Application***
Events are independent if the probability of A and B occurring together is the product of their individual probabilities. In independent events, the outcome of the first event does not change the sample space for the outcomes of subsequent events.	Test for independence using the definition of independent events.	State problems' independence and dependence contextually.
G.S.CP.A.3 Determine conditional probabilities and interpret independence by analyzing conditional probability.		
Conceptual Understanding	***Procedural Fluency***	***Application***
Conditional probability is the likelihood that an event occurs given that another event has occurred. Events are independent if the conditional probability of an event is equal to the probability of the event without the condition.	Calculate conditional probabilities. Relate conditional probability to relative frequency tables and/or tree diagrams. Use conditional probabilities to determine whether events are independent.	Contextually interpret probability of events.
G.S.CP.A.4 Construct and interpret two-way frequency tables of data. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.		
Conceptual Understanding	***Procedural Fluency***	***Application***
Two-way frequency tables describe a sample space of a population and can be used to determine conditional probability.	Set up and calculate conditional probability using two-way frequency tables for various data.	Collect sample data from a real-world situation in order to examine conditional probabilities and independence of events. Interpret and make sense of these in context of the situation. Use the data to make predictions about the whole population.

G.S.CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Conceptual Understanding	***Procedural Fluency***	***Application***
When events are dependent, there is an association between the variables. Distinguish between association and causality.	Use conditional probability to make decisions and justify claims of relationships to contextual situations.	Interpret conditional probability and independence across a variety of situations.

Enacting the Mathematical Practices - Evidence of Students Engaging in the Practices

1. **Make sense of problems and persevere in solving them.**
    - Learners must be challenged to develop deep understanding through exploring a range of tasks that require problem solving.
    - Make sense of formulas and the relationships among them.
2. **Reason abstractly and quantitatively.**
    - Justifying formulas will move learners from concrete to abstract thinking.
    - Use formulas strategically in calculations.
3. **Construct viable arguments and critique the reasoning of others.**
    - Learners justify their process for solutions and connect ideas of independence and disjoint sets.
4. **Model with mathematics.**
    - Learners may model problems and look for structure with Venn diagrams, tables, or other methods in order to solve problems and make generalizations.
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
    - Through precise descriptions, learners attend to problem contexts and ensure the appropriate interpretation of results.
    - Precisely use set notation to describe compound and conditional probability.
7. **Look for and make use of structure.**
    - Learners focus on the differences and similarities of different sets' relationships.
    - Learners attend to the structure of the set, definitions, and solutions to verify the results of the relationship and notations of the questions.
    - Learners use structure of different representations including Venn Diagrams and tables to develop rules.
8. **Look for and express regularity in repeated reasoning.**
    - Learners develop formulas for the Addition Rule and Multiplication Rule through the exploration of probability relationships.

Vertical and Horizontal Coherence and Learning Progressions

Previous Learning Connections	***Current Learning Connections***	***Future Learning Connections***
In previous years, learners used sample spaces to represent compound events in organized lists, tables, and tree diagrams. Learners are initially introduced to probability in 7th grade. They have investigated chance processes and developed probability models using experimental and theoretical probability. **(7.SP.5, 7.SP.6, 7.SP.7, 7.SP.8)**	Learners will use their knowledge of conditional probability and their skills of determining conditional probability to make decisions for real world situations. They will also expand the knowledge of this cluster to learn specific rules such as the Addition Rule. This knowledge will lead into permutations and combinations.	Learners will extend their learning to develop and make sense of the Multiplication Rule and Addition Rule. Future learning such as binomial distribution and statistical significance build upon conditional probability. Other applications are found in calculus, statistics, engineering, and the sciences. Many careers including actuaries, industrial engineers, statisticians, and production managers will frequently use statistics concepts including conditional

<table>
<tr><td></td><td></td><td>probability.
(S.CP.B)</td></tr>
<tr><td colspan="3">Vocabulary (key terms and definitions)</td></tr>
<tr><td>● subset
● union
● intersection
● complement
● set notation</td><td>● Venn diagram
● sample space
● event
● outcome
● conditional probability</td><td>● two-way frequency table
● independent
● dependent
● compound event
● disjoint sets</td></tr>
<tr><td colspan="3">Relevance, Explanations, and Examples:</td></tr>
<tr><td colspan="3">Relating two-way frequency tables to geometric area models can be an important connection for learners.

Learners can feel confident in their understanding before they have a true conceptual understanding. The Monty Hall problem is an example of a surprising result of conditional probability and can be used to challenge learners to think deeply about independence and conditional probability.</td></tr>
<tr><td colspan="3">Achievement Level Descriptors</td></tr>
<tr><td colspan="3">Cluster: Understand independence and conditional probability and use them to interpret data</td></tr>
<tr><td rowspan="4">Concepts and Procedures</td><td colspan="2">Level 1:</td></tr>
<tr><td colspan="2">Level 2:</td></tr>
<tr><td colspan="2">Level 3:</td></tr>
<tr><td colspan="2">Level 4:</td></tr>
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